# VWAP Cost Excluding Own Trades

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VWAP (Volume-Weighted Average Price) Cost, first introduced by Berkowitz, Logue, and Noser [1988], is a widely used measure for execution quality. For example, Schwartz and Steil [2002] find that Chief Investment Officers rank VWAP performance well above other criteria for evaluating how well their traders handle their orders.

The idea behind VWAP Cost is quite simple: comparing the trader's own execution price against the market "average" price during the trader's trading window. It is very much the same philosophy behind comparing a mutual fund's performance against a market index return (e.g., the S&P 500). A nice analogy can be drawn between the VWAP in the context of trading and the index in the context of investing. VWAP Cost (multiplied by -1) is like the *alpha* in investment performance measurement. Since we lack a counterpart to the CAPM for trading, we simply benchmark a trader against the average, i.e., the VWAP. Trading to achieve the VWAP is like managing an index fund to achieve the market index return. If we call indexing "passive investing," then VWAP trading might be called "passive trading." Just as indexing has become increasingly popular over the last several decades (partly inspired by extensive academic research), VWAP trading has become more popular among practitioners in recent years. Conversations with Japanese and Hong Kong traders who trade U.S. equity show that they are especially happy about this development. Now they can put trading on "autopilot" and go to sleep gracefully, knowing that they will be able to achieve the market average when they awaken.

"VWAP trading" (trading to achieve VWAP as the execution price) and "VWAP Cost" (using VWAP as a benchmark price to measure an active trader's execution quality) are very different concepts. Simply achieving the VWAP does not mean that the trader (broker) has abnormal skill, just as an index fund manager does not have abnormal skill. A trader (broker) has abnormal skill only if he can consistently beat the VWAP, in other words, a consistently negative and significant VWAP Cost. See Lert [2001] and Hu [2005] for more detailed discussions regarding using VWAP as a benchmark to measure execution quality.

Many have criticized the VWAP Cost measure. See, for example, Schwartz and Steil [2002]. One of the most frequent criticisms is that VWAP Cost includes the trader or institutional investor's own trades in the benchmark price. This is not a new problem. Note that we have a similar concern when we evaluate a mutual fund's performance using a market index as the benchmark, because stocks held by a mutual fund are also part of the market index.

This article clarifies related issues by deriving a simple mathematical relation between

VWAP Cost including versus excluding own trades. This result enables computation of VWAP Cost excluding own trades without having to explicitly identify the trader's own trades in intraday market data. This is useful because it is extremely difficult, if not impossible, to identify a particular trader's trades in intraday market data.

## NOTATION

First I define some notations:

$P_{own}$ : Trader's own execution price. It is also volume-weighted.
$V_{own}$ : Trader's own volume—number of shares traded.
$VWAP_{mkt}$ : Volume-Weighted Average Price of all trades in the market.
$V_{mkt}$ : Total market volume.
$VWAP_{other}$ : Volume-Weighted Average Price of all other traders' trades, excluding the trader's own trades.
$V_{other}$ : Total trading volume by other traders, excluding the trader's own trades.
$f \equiv \frac{V_{own}}{V_{mkt}}$ : Trader's market share: the trader's own volume as a fraction of total market volume. By definition, $0 \le f \le 1$.

## $VWAP_{mkt}$ AS A WEIGHTED AVERAGE OF $P_{own}$ AND $VWAP_{other}$

It is useful to think of $VWAP_{mkt}$ (the total market VWAP) as a weighted average/convex combination of $P_{own}$ (trader's own execution price) and $VWAP_{other}$ (VWAP of other traders' trades), with their respective market shares as weights. We know the following is true:

$$\begin{cases} V_{mkt} = V_{own} + V_{other} \\ VWAP_{mkt} \cdot V_{mkt} = P_{own} \cdot V_{own} + VWAP_{other} \cdot V_{other} \end{cases}$$

In other words, the total market share volume should equal the sum of the trader's own shares traded and all other traders' share volume. Similarly, the total market dollar volume should equal the sum of the trader's own dollar volume and all other traders' dollar volume. Hence, the following must be true:

$$\begin{aligned} VWAP_{mkt} &= P_{own} \cdot \frac{V_{own}}{V_{mkt}} + VWAP_{other} \cdot \frac{V_{other}}{V_{mkt}} \\ &= P_{own} \cdot f + VWAP_{other} \cdot (1 - f) \end{aligned}$$

Therefore, $VWAP_{mkt}$ is always somewhere between $P_{own}$ and $VWAP_{other}$. $VWAP_{mkt}$ approaches $P_{own}$ as the trader's market share, $f$, approaches 1. $VWAP_{mkt}$ approaches $VWAP_{other}$ as the trader's market share, $f$, approaches 0.

## DEFINE *VWAP* COST INCLUDING VERSUS EXCLUDING OWN TRADES

For a *buy* order (add a minus sign in front of the right-hand side for a *sell* order), we define:

$$VWAP\ Cost \equiv \frac{P_{own} - VWAP_{mkt}}{P_{own}}$$

Note that the benchmark price, $VWAP_{mkt}$, includes the trader's own trades. Alternatively, we can define:

$$VWAP\ Cost\ Excluding\ Own\ Trades \equiv \frac{P_{own} - VWAP_{other}}{P_{own}}$$

VWAP Cost can be thought of as execution quality compared to the *market average*, while VWAP Cost Excluding Own Trades can be thought of as execution quality compared to the *average of everyone else*. The only difference is that the benchmark price, $VWAP_{other}$, excludes the trader's own trades. However, $VWAP_{other}$ is not directly observable. It is extremely difficult, if not impossible, to identify a particular trader's trades in intraday market data (e.g., the NYSE TAQ).

Note that I use the trader's own execution price, $P_{own}$, in the denominator. Sometimes the benchmark price, $VWAP_{mkt}$ or $VWAP_{other}$, is used in the denominator. Both make sense and it may not make a big difference empirically. I use this definition for the following reasons:

1) It makes it easier to compare results obtained using different benchmark prices, because they all have the same denominator.
2) It makes it more convenient to compute the corresponding dollar trading cost (simply multiply the trading cost measure by the dollar principal traded). For the same reason, it also makes it easier to compute aggregate trading costs.
3) The mathematical relation between VWAP Cost and VWAP Cost Excluding Own Trades will hold *exactly* only if we use the trader's own execution price in the denominator. If we use the benchmark

price in the denominator, the relation will only hold *approximately*.

## RELATION BETWEEN *VWAP* COST INCLUDING VERSUS EXCLUDING OWN TRADES

There is a *simple* mathematical relation between VWAP Cost (including own trades) and VWAP Cost Excluding Own Trades:

$$VWAP\ Cost\ Excluding\ Own\ Trades = VWAP\ Cost \cdot \left(\frac{1}{1-f}\right)$$

**Proof:**

Following from definitions:

$$\frac{VWAP\ Cost\ Excluding\ Own\ Trades}{VWAP\ Cost} = \frac{P_{own} - VWAP_{other}}{P_{own} - VWAP_{mkt}}$$

we also have:

$$\begin{cases} V_{mkt} = V_{own} + V_{other} \\ VWAP_{mkt} \cdot V_{mkt} = P_{own} \cdot V_{own} + VWAP_{other} \cdot V_{other} \end{cases}$$

$$\Rightarrow \quad VWAP_{other} = \frac{VWAP_{mkt} \cdot V_{mkt} - P_{own} \cdot V_{own}}{V_{other}}$$

$$\begin{aligned} \Rightarrow P_{own} - VWAP_{other} &= P_{own} - \frac{VWAP_{mkt} \cdot V_{mkt} - P_{own} \cdot V_{own}}{V_{other}} \\ &= \frac{P_{own} \cdot V_{other} - VWAP_{mkt} \cdot V_{mkt} + P_{own} \cdot V_{own}}{V_{other}} \\ &= \frac{P_{own} \cdot (V_{other} + V_{own}) - VWAP_{mkt} \cdot V_{mkt}}{V_{other}} \\ &= (P_{own} - VWAP_{mkt}) \cdot \frac{V_{mkt}}{V_{other}} \end{aligned}$$

$$\Rightarrow \quad \frac{P_{own} - VWAP_{other}}{P_{own} - VWAP_{mkt}} = \frac{V_{mkt}}{V_{other}} = \frac{1}{\frac{V_{other}}{V_{mkt}}} = \frac{1}{1 - \frac{V_{own}}{V_{mkt}}} = \frac{1}{1-f}$$

Therefore,

$$\begin{aligned} \frac{VWAP\ Cost\ Excluding\ Own\ Trades}{VWAP\ Cost} &= \frac{P_{own} - VWAP_{other}}{P_{own} - VWAP_{mkt}} \\ &= \frac{1}{1-f} \end{aligned}$$

QED.

## ILLUSTRATION OF THE RELATION

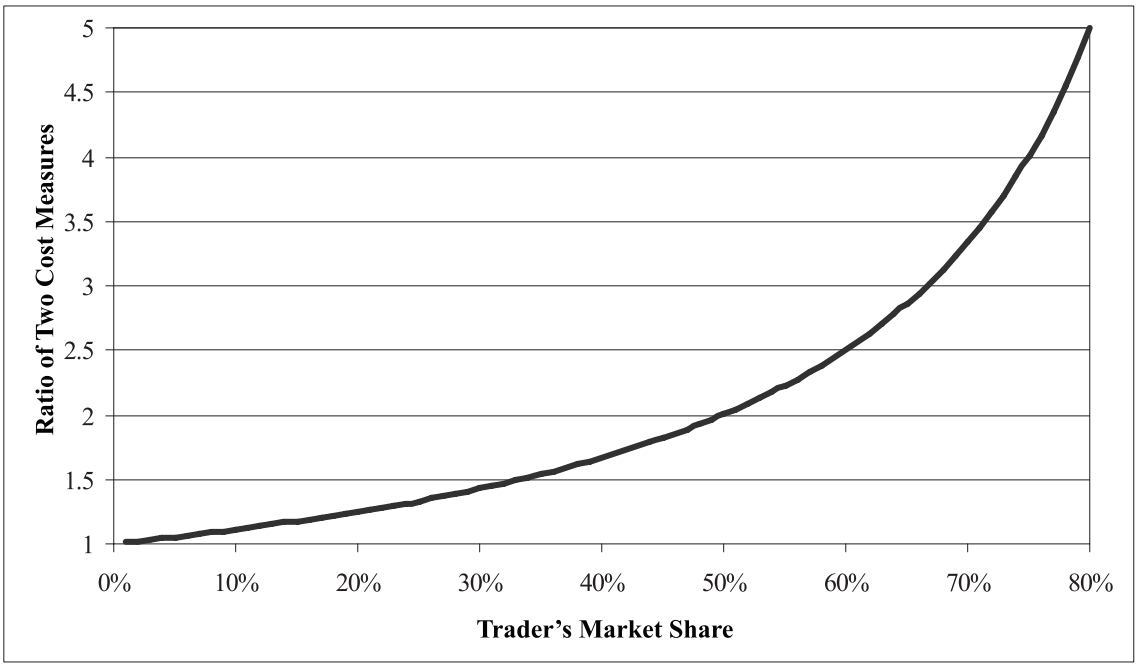
The above relation is interesting. It means that, compared to VWAP Cost, VWAP Cost Excluding Own Trades always overstates gains or losses. The ratio of the two cost measures is a monotonically increasing function of the trader's own market share, *f*. The following Exhibit 1, illustrates this relation.

As one can see, when the trader's market share, *f*, is small, the difference between the two measures is negligible. The ratio of the two cost measures increases at an increasing speed as the trader's market share increases. This can be seen in the following two Exhibits 2 and 3, one illustrating the relation over reasonable range of the market share parameter (less than 80%), and the other plotting the entire parameter range.

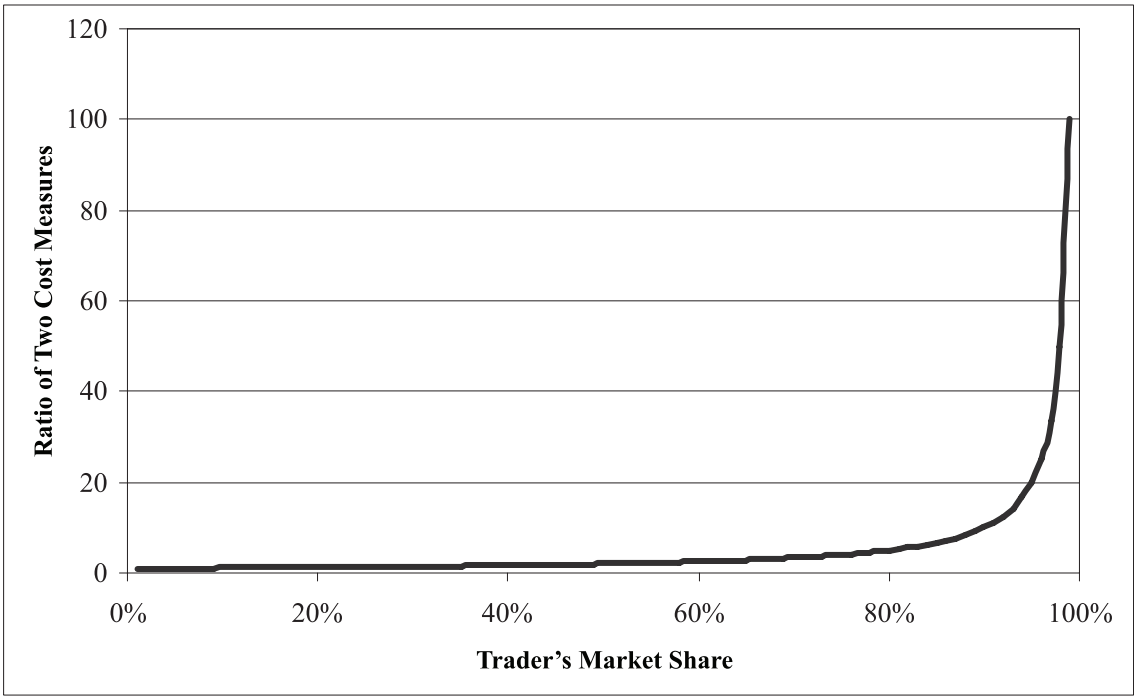
## EXHIBIT 1

| Trader's Market Share $(f)$ | Ratio of Two Cost Measures $\left(\frac{1}{1-f}\right)$ |
|---|---|
| 1% | 1.01 |
| 2% | 1.02 |
| 5% | 1.05 |
| 10% | 1.11 |
| 20% | 1.25 |
| 30% | 1.43 |
| 50% | 2 |
| 80% | 5 |
| 99% | 100 |

## EXHIBIT 2




## EXHIBIT 3



## CONCLUSION

VWAP Cost, though widely used, is frequently criticized for including the trader's own trades in the benchmark price. I derive a simple mathematical relation between VWAP Cost including versus excluding own trades. The ratio of the two cost measures is shown to be a direct function of only one parameter: the trader's market share.

One can use this result to compute VWAP Cost Excluding Own Trades without having to explicitly identify the trader's own trades in intraday market data. This is useful in practice because it is extremely difficult, if not impossible, to identify a particular trader's trades in intraday market data.

## ENDNOTE

For helpful comments and discussions, I thank Ginger Meng, Erik Sirri, George Sofianos, and my former colleagues at Fidelity Management & Research Company (FMRCO), Global Equity Trading, Trading Techniques & Measurement Group, especially Ani Chitaley, Arun Gorur, Peter Lert, Mark Matthews, and Don Slowik. All remaining errors and omissions are my own.

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